

THE NATURE AND GEOMETRY OF SPACE.

I HAVE to thank Mr. Turner for his courteous criticisms on my article about "Space". But they appear to me to rest mainly on misunderstandings of the position that I was trying to maintain and on a failure to notice certain distinctions and definitions which I laid down. Mr. Turner also argues from some premises which seem to me to be false. It will therefore be worth while to make a short reply in the hope of reaching a better understanding. In quoting Mr. Turner's criticisms I will refer to them as T. and give the page in *MIND* No. 98 from which the quotation is made.

(1) 'It would seem to follow that each "private" space must contain its own "private" matter; we cannot restrict this "privacy" . . . to space . . .' (T. 223). . . . 'Mr. Broad . . . later qualifies his position by the assertion that we do not know whether such (private perceptual) spaces be possible.' (T. 224.) My position, which I thought I made clear in the article, is this: (a) *If* there be genuine private spaces there will be private matter in them. (b) If there be physical space and physical matter then the physical matter is not in any private space. (c) The reason why it is doubtful whether there are private spaces with private matter is not that the *privacy* of our sense-data seems doubtful nor that it is doubtful that our sense-data stand in *spatial* relations to each other. The question is whether a private world consisting of sense-data in spatial relations can also be regarded as a space containing matter, when we define the difference between space and matter as I define it in the article. It is curious that Mr. Turner never explicitly mentions the distinction which I laid down between space and matter and tried to defend; indeed it is clear by implication that he ignores it and takes a different distinction which I do not accept. Thus he argues that material unextended points are impossible because *material* and *unextended* exclude each other by definition (T. 227). Now they may exclude each other on Mr. Turner's definition, but they do not do so on mine. On my definition geometrical points stand in timeless spatial relations to each other; whilst material points stand in spatial relations which may alter, and these relations are themselves dependent upon two others—(a) the relation of being *at* a geometrical point at a certain moment and (b) that of two geometrical points to each other. With this definition there is no objection to material unextended points.

Mr. Turner's real objection to unextended material points is no

doubt contained in his footnote (T. 228) where he asks: 'Can a selection of unextended points be an extended straight line?' I should reply: Most certainly. A straight line is a series of unextended points ordered by a certain relation. Similarly an extended bit of matter can be regarded as a set of unextended material points ordered by certain relations. I do not think that any one acquainted with the modern mathematical theory of the continuum will quarrel with these views.

(2) . . . 'What grounds do we possess for our knowledge that this space, which I add to *my* experience is identical with the space that you (and others) add to your (and their) experience? Why, if perceptual spaces are private, should not thought-constructed spaces be private also' (T. 224). I argued in my paper that physical space was constructed to deal, not merely with *my* sense-data, but with all the sense-data of all normal waking persons. Assuming that it is possible for us to communicate with other people and to learn about their experiences there is nothing impossible in such a construction. And if it be possible at all it will naturally not be private to any one person. I did not indeed deal with the limitations and implications of intersubjective intercourse, but I would point out that exactly the same problem would arise for Mr. Turner. He rejects what he calls 'private space theories' and prefers the view that 'all appearance is partial reality' (T. 223. See especially note 4). How far this view is incompatible with 'private space theories' I could not undertake to say unless I knew much more accurately than I do in what sense Mr. Turner interprets it and them. But, at any rate, Mr. Turner has to 'supplement' his appearances 'from regions outside themselves'. He can only learn about the latter from descriptions given by other people, and so any difficulty about the reliability of intersubjective intercourse and the possibility of thought being as private as sensation affects him as much as it affects me. I may as well at once reply to 'the popular opinion that our "thoughts" are essentially private' (T. 224. Note 7) which Mr. Turner considers important in this connexion. My thought of the number 2 is an event in my mind with a certain place in my history; it differs from Mr. Turner's thought of the number 2 which is an event in his mind. Both these thoughts are thus private. But their object, the number 2, is not private. Similarly physical space is not private, though my act of thinking about space is private to me and Mr. Turner's act of thinking about space is private to him.

(3) I never denied for a moment, as Mr. Turner seems to think (T. 225), that 'the content of perception (spatial relations) and the object of thought (space) have some common¹ real element'. Nor

¹ I do not understand the force of the word *real* here. If Mr. Turner means that there must be particular existent parts common to private and physical space I disagree. But this is not necessary in order that the adjective *spatial* may reasonably be applied to both.

did I suggest for an instant that what we perceive is spatial *relations* and nothing else, and that physical space does not involve spatial relations. What we perceive at any moment with any sense is a complex whole in which we can distinguish on analysis extended terms and spatial relations. These relations have certain logical properties and are the clue to all that we assert about physical space. The logical properties that we ascribe to the relations of points in physical space are not indeed *identical* with those which are possessed by the relations which we find in the object of sense-perception; if they were physical space would not do the work for which it is constructed, *viz.* the correlation of *all* sense-data in *all* normal waking people. But there is enough analogy to make it perfectly reasonable to call both kinds of relation spatial. And, since in my view the points, lines, etc., of physical space are expressly constructed to deal with sense-data and their relations, I cannot see much relevance in Mr. Turner's remark (T. 226) that 'unless these definitive terms have some reference ultimately to the content of perceptual experience, they become wholly void of meaning'.

(4) It remains for me to notice an argument of Mr. Turner's which I do not profess to understand. He says (T. 225) that if we treat space as I treated it in my article 'we must logically take the same view of force, time, mass, truth, duty, and every other ultimate scientific and moral concept alike'. This, Mr. Turner thinks, is a 'serious dilemma'. I can only reply by proposing the following dilemma to him. Either duty, truth, etc., are very much like space or they are not. If they are there can be no *objection* to treating them in the same way as space, and if they are not there can be no *need* to treat them in the same way as space.

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